USN | | | | | | | | | | 10MAT31

Third Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics - III

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Obtain Fourier series for the function f(x) given by

$$f(x) = \begin{cases} 1+\dfrac{2x}{\pi}, & -\pi \le x \le 0 \\ 1-\dfrac{2x}{\pi}, & 0 \le x \le \pi \end{cases}.$$

Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots=\frac{\pi^2}{8}$. **(06 Marks)**

b. Obtain Fourier half range Cosine series for the function f(x) = x sin x in (0, π). Hence show that $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\ldots..\infty=\frac{\pi-2}{4}$. **(07 Marks)**

c. Obtain the constant term and the co-efficient of the first sine and cosine terms in the Fourier series of f(x) as given in the following table. **(07 Marks)**

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

2 a. Find the Fourier transform of $e^{-a^2x^2}$, a < 0. Hence deduce that $e^{-x^2/2}$ is self reciprocal in respect of Fourier transform. **(06 Marks)**

b. Find the Fourier sine transform of $e^{-|x|}$. Hence show that

$$\int_0^\infty \frac{x\sin mx}{1+x^2}dx=\frac{\pi e^{-m}}{2},\ m>0.$$ **(07 Marks)**

c. Find the Fourier Cosine transform of $f(x)=\frac{1}{1+x^2}$. **(07 Marks)**

3 a. Obtain various possible solutions of the one dimensional Heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables. **(06 Marks)**

b. Obtain the D'Alembert's solution of the wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$. Subject to the conditions u(x, 0) = f(x) and $\frac{\partial u}{\partial t}(x,0)=0$. **(07 Marks)**

c. Obtain various possible solutions of the two dimensional Laplace equation $u_{xx}+u_{yy}=0$ by the method of separation of variables. **(07 Marks)**

4 a. Fit a parabola $y = ax^2 + bx + c$ to the following data : **(06 Marks)**

x	0	1	2	3	4	5
y	1	3	7	13	21	31

b. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 5,760 to invest and has space for at most 20 items. A fan and sewing machine cost Rs 360 and Rs 240 respectively. He can sell a fan at a profit of Rs 22 and sewing machine at a profit of Rs 18. Assuming that he can sell whatever he buys, how should he invest his money in order to maximize his profit? Translate the problem into LPP and solve it graphically. **(07 Marks)**

c. Use Simplex method to solve the following LPP

Minimize $Z = x_1 - 3x_2 + 3x_3$

Subject to $3x_1 - x_2 + 2x_3 \le 7$

$2x_1 + 4x_2 \ge -12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

$x_1, x_2, x_3 \ge 0.$ **(07 Marks)**

PART – B

5 a. Using Newton – Raphson method, find the value of $\sqrt[3]{18}$ correct to 2 decimals, assuming 2.5 as the initial approximation. **(06 Marks)**

b. Apply Gauss – Seidal iteration method to solve the following equations :

$3x + 20y - z = -18$; $2x - 3y + 20z = 25$; $20x + y - 2z = 17$. **(07 Marks)**

c. Find the largest Eigen - value and the corresponding Eigen - vector for the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$ with initial approximation $[1\ 1\ 0]^T$. **(07 Marks)**

6 a. Determine f(x) as a polynomial in x for the following data by using Newton's divided difference formula. **(06 Marks)**

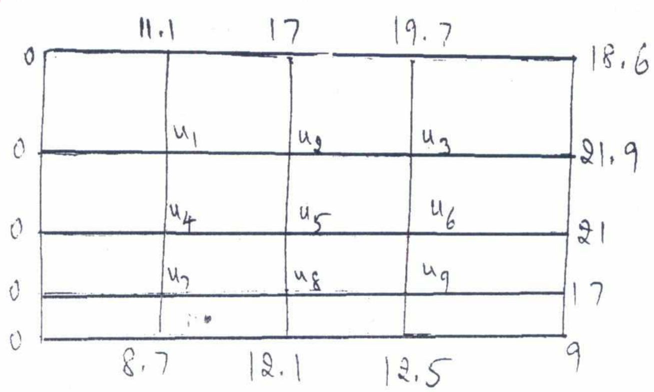
x	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

b. From the data given in the following table, find the number of students who obtained i) less than 45 marks and ii) between 40 and 45 marks. **(07 Marks)**

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

c. Evaluate $\int_4^{5.2} \log_e x \, dx$ by Weddle's rule. **(07 Marks)**

7 a. Solve the Laplace equation $u_{xx} + u_{yy} = 0$, given that the boundary values for the following square mash. **(06 Marks)**



b. Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$, taking h = 1 upto t = 1.25. The boundary conditions are $u(0,t) = u(5, t) = 0$, $u_t(x, 0) = 0$ and $u(x, 0) = x^2(5-x)$. **(07 Marks)**

c. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in $0 < x < 5$, $t \geq 0$, given that $u(x, 0) = 20$, $u(0, t) = 0$, $u(5, t) = 100$. Compute u for the time – step with h = 1 by Crank – Nicholson method. **(07 Marks)**

8 a. Find the Z – transform of the following :
i) $(n + 1)^2$ ii) $\sin(3n + 5)$ iii) n_{c_p} $(0 \leq p \leq n)$. **(06 Marks)**

b. If $u(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$. Find u_0, u_1, u_2, u_3. **(07 Marks)**

c. Solve $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ with $y_0 = 0$, $y_1 = 1$, using Z - transforms. **(07 Marks)**

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10ME/AU32B

Third Semester B.E. Degree Examination, June/July 2017
Mechanical Measurements & Metrology

Time: 3 hrs. Max. Marks:100

Note: *Answer FIVE full questions, selecting at least TWO questions from each part.*

PART – A

1 a. Define measurement. List the objectives of metrology. **(06 Marks)**

b. Describe with neat sketch,
(i) International prototype meter.
(ii) Imperial standard yard. **(10 Marks)**

c. Write a brief note on slip gauges. **(04 Marks)**

2 a. Determine the tolerance on the hole and shaft for a precision running fit designated by $50H_7g_6$:
Given : (i) 50 mm lies between 30 – 50 mm
(ii) $i\,(\text{microns}) = 0.45D^{\frac{1}{3}} + 0.001D$
(iii) Fundamental deviation for 'g' shaft $= -2.5D^{0.34}$
(iv) IT7 = 16i
(v) IT6 = 10i
State the actual maximum and minimum sizes of the hole and shaft and maximum and minimum clearances. **(12 Marks)**

b. Explain hole basin and shaft basin system. **(04 Marks)**

c. Explain inter changeability and wear allowance. **(04 Marks)**

3 a. With the help of a sketch, explain sigma comparators. **(10 Marks)**

b. Explain with a neat sketch, the construction and working of LVDT. **(10 Marks)**

4 a. With a neat sketch, explain auto collimeter. **(12 Marks)**

b. Explain three wire method of measuring effective diameter of screw thread. **(08 Marks)**

PART – B

5 a. With a block diagram, explain the generalized measurement system with example. **(10 Marks)**

b. Define the following terms:
(i) Accuracy (ii) Repeatability (iii) Sensitivity (iv) Calibration
(v) Hysterisis. **(10 Marks)**

6 a. Explain with a neat sketch, the working principle of CRO. **(10 Marks)**

b. With a neat sketch, the working of X-Y plotters. **(10 Marks)**

7 a. With a neat sketch, explain hydraulic dynamometer and mention its advantages and disadvantages. **(10 Marks)**

b. With a neat sketch, explain the working principle of proving ring. **(10 Marks)**

8 a. With a neat sketch, explain the working of optical pyrometer. **(12 Marks)**

b. What is thermo couple? State the laws of thermo couple. **(08 Marks)**

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10ME/AU33

Third Semester B.E. Degree Examination, June/July 2017
Basic Thermodynamics

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer FIVE full questions, selecting at least TWO questions from each part.***
2. ***Use of thermodynamic data hand book is permitted.***

PART – A

1 a. What is meant by thermodynamic equilibrium? Explain mechanical, chemical and thermal equilibrium. **(08 Marks)**

b. Distinguish between:
   (i) Macroscopic and microscopic approaches.
   (ii) Point and path functions.
   (iii) Cyclic and non cyclic process. **(06 Marks)**

c. The temperature on a thermometric scale is defined in terms of a property K by the relation $t = M\ln(K) + N$, where M and N are constants. The values of K are found to be 1.83 and 6.78 at Ice and steam points respectively. Determine the temperature corresponding to a value of K = 2.42 on the thermometer. **(06 Marks)**

2 a. Define work according to:
   (i) Mechanics.
   (ii) Thermodynamics. **(04 Marks)**

b. State the conditions to be satisfied for displacement work. Derive an expressions for displacement work in,
   (i) Isothermal process.
   (ii) Polytropic process. **(10 Marks)**

c. A spherical balloon has an initial diameter of 25 cm and contain air at 1.2 bar. Because of heating, the diameter of balloon increases to 30 cm and during the heating process, the pressure is found to be proportional to the diameter. Calculate the work done during the process. **(06 Marks)**

3 a. Prove that energy (E) is a property of a system. **(08 Marks)**

b. Derive steady flow energy equation with usual notations. **(06 Marks)**

c. A slow chemical reaction takes place in a fluid at constant pressure of 0.1 MPa. The fluid is surrounded by a perfect heat insulator during the reaction which begins at state 1 and ends at state 2. The insulation is removed and 150 KJ of heat flow to the surrounding as the fluid goes to state 3. The following data are observed for the fluid at states 1, 2 and 3.

State	Volume (m^3)	Temperature (°C)
1	0.003	20
2	0.3	70
3	0.06	20

Find the energy for this fluid system at states 2 and 3, if the energy at state 1 is zero. **(06 Marks)**

4 a. Show that COP of the heat pump is greater than COP of a refrigerator by unity. **(06 Marks)**
b. State and prove Carnot's theorem. **(06 Marks)**
c. An ice plant working on a reversed Carnot cycle; heat pump produces 15 tonnes of ice per day. The ice is formed from water at 0° C and formed ice is maintained at 0° C. The heat is rejected to the atmosphere at 25°C. The heat pump used to run the plant is coupled to Carnot engine which absorbs heat from the source which is maintained at 220°C by burning fuel of 44.5 MJ/kg calorific value and rejecting heat to atmosphere.
Determine (i) Power developed by the engine. (ii) Fuel consumed / hr.
Take enthalpy of fusion of ice = 334.5 KJ/kg. **(08 Marks)**

PART – B

5 a. With usual notations, explain Clausius theorem. **(08 Marks)**
b. Explain principle of increase of entropy. **(04 Marks)**
c. One kg of ice at -5°C is exposed to the atmosphere which is at 20°C. The ice melts and comes into thermal equilibrium with the atmosphere. Determine the entropy increase of the universe.
Take C_P of ice 2.093 KJ/kg K and
Latent heat of fusion of ice 333.3 KJ/kg. **(08 Marks)**

6 a. With the help of PV-diagram, explain the various regions of a pure substance. **(06 Marks)**
b. Sketch and explain combined separating and throttling calorimeter. **(06 Marks)**
c. Steam at 10 bar and 230°C is cooled under constant pressure until it becomes 0.85 dry. Using steam tables find the work done, change in enthalpy, heat transferred and change in entropy. **(08 Marks)**

7 a. Define Ideal gas. **(02 Marks)**
b. Show that the change in entropy for ideal gas is given by the expression:
$$(S_2 - S_1) = C_P \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$ **(10 Marks)**
c. An ideal gas cycle consisting of three processes uses Argon (molecular weight = 40) as a working substance. Process 1 – 2 is reversible adiabatic process from 0.14 m^3, 700 KPa and 280°C to 0.056 m^3. The process 2 – 3 is a reversible isothermal process. Process 3 – 1 is an isobaric process. Sketch the cycle on P – V and T – S diagrams and find,
(i) Work transfer in process 1 – 2
(ii) Work transfer in process 2 – 3
(iii) Net work output from the cycle.
Assume $\gamma = 1.67$ **(08 Marks)**

8 a. Distinguish between ideal and real gases. **(04 Marks)**
b. Explain (i) Compressibility factor and
(ii) Compressibility chart. **(04 Marks)**
c. Define Dalton's law of partial pressure. **(04 Marks)**
d. Find the gas constant and apparent molar mass of a mixture of 2 kg O_2 and 3 kg N_2, given that universal gas constant is 8314.3 J/kgmoleK. Molar masses of O_2 and N_2 are respectively 32 and 28. **(08 Marks)**

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Third Semester B.E. Degree Examination, June/July 2017

Mechanics of Materials

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. List and explain the mechanical properties of engineering materials. **(10 Marks)**

b. A round stepped bar is subjected to an axial force of 30 kN. Diameter and length of first portion are 40 mm and 200 mm respectively and those of second portion are 20 mm and 100 mm respectively. Determine change in deformation when a uniform bar with same volume and length as that of stepped bar is subjected to 30 kN. Take E = 200 GPa. **(10 Marks)**

2 a. Show the relation between Young's modulus and modulus of rigidity. **(10 Marks)**

b. A compound bar consisting of steel, Bronze and aluminium bars connected in series is held between two supports as shown in Fig.Q2(b). When the temperature of the compound bar is increased by 50°C, determine the stresses induced in each bar. Consider the two cases: i) Rigid supports and ii) Supports yield by 0.5 mm. Take $\alpha_s = 12\times10^{-6}/°C$, $\alpha_B = 19\times10^{-6}/°C$, $\alpha_{AL} = 22\times10^{-6}/°C$, E_s = 200 GPa, E_B = 83 GPa and E_{AL} = 70 GPa.

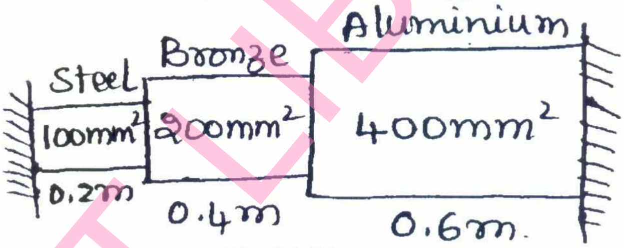

Fig.Q2(b) **(10 Marks)**

3 a. Explain : i) Principal planes and principal stresses and ii) Maximum and minimum shear stresses with respect to compound stress. **(06 Marks)**

b. Describe the construction of Mohr's circle for plane stress. **(06 Marks)**

c. A point in a beam is subjected to maximum tensile stress 110 MPa and shear stress 30 MPa. Find the magnitudes and directions of principal stresses. If the point in the beam is in the compression zone under the same magnitude of bending stress and shear stress. Find the magnitudes of principal stresses and their directions. **(08 Marks)**

4 a. Explain the concept of circumferential stress and longitudinal stress corresponding to thin cylinders. **(10 Marks)**

b. A cylindrical pressure vessel of 1 meter inner diameter and 1.5 meters long is subjected to an internal pressure P, thickness of the cylinder wall is 15 mm. Taking allowable stress for cylinder material as 90 MPa. Determine: i) Magnitude of maximum internal pressure P that the pressure vessel can with stand and ii) Change in dimensions. Take E = 200 GPa and $\gamma = 0.3$. **(10 Marks)**

PART – B

5 a. Define and explain the following terms:
i) Shear force ii) Bending moment
iii) Shear force iv) Bending moment diagram **(06 Marks)**

b. Define and explain the following types of load:
i) Concentrated load
ii) Uniformly distributed load
iii) Uniformly varying load **(06 Marks)**

c. A simply supported beam of length 6m, carries point load of 3 kN and 6 kN at distances of 2 m and 4 m from the left end. Draw the shear force and bending moment diagrams for the beam. **(08 Marks)**

6 a. What do you mean by 'simple bending'? What are the assumptions made in the theory of simple bending? **(07 Marks)**

b. A cantilever of length 2 meter fails when a load of 2 kN is applied at the free end. If the section of the beam is 40 mm × 60 mm, find the stress at the failure. **(05 Marks)**

c. An I-section beam 350 mm × 150 mm has a web thickness of 10 mm and a flange thickness of 20 mm. If the shear force acting on the section is 40 kN, find the maximum shear stress developed in the I-section. **(08 Marks)**

7 a. Derive an equation for deflection. **(10 Marks)**

b. What is a Macaulay's methods? Where is it used? **(04 Marks)**

c. A 2 meters long cantilever is subjected to UDL of 10 kN/m throughout its length and is vertically downward point load 20 kN at its free end. Taking E = 200 GPa and maximum deflection as 0.3 mm determine the width and depth of rectangular section. Depth of the section is twice the width. **(06 Marks)**

8 a. Derive the relation for a circular shaft when subjected to torsion as given by $\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$.
Also list out the assumptions made while deriving the relation. **(10 Marks)**

b. Derive an expression for the Euler's crippling load for a long column when both the ends of the column are hinged. **(10 Marks)**

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Third Semester B.E. Degree Examination, June/July 2017
Manufacturing Process – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Define the casting process. Explain the different steps involved in casting process. **(08 Marks)**
b. What is pattern? Explain briefly any two types of pattern allowances. **(06 Marks)**
c. Write a Note on Binders and Additives used in moulding. **(06 Marks)**

2 a. With a neat figure, explain the Terminologies of sand mould. **(08 Marks)**
b. What are the desirable properties of moulding sand? **(06 Marks)**
c. Mention the various casting defects. Explain any two types of defects. **(06 Marks)**

3 a. With a neat sketch, explain the investment moulding process. Mention its advantages. **(10 Marks)**
b. Define the "Die casting". With a neat sketch, Explain the Hot chamber die casting process. **(10 Marks)**

4 a. With a neat sketch, explain the working principle of electric Resistance Furnace. **(10 Marks)**
b. Explain the construction and working principle of a cupola Furnace, with a sketch. **(10 Marks)**

PART – B

5 a. Define welding process. What are the advantages and limitations of welding process? List the industrial applications of welding. **(10 Marks)**
b. With a neat sketch, explain the Tungsten Inert gas welding (TIG) with advantages. **(10 Marks)**

6 a. With a neat sketch, explain the Resistance Butt welding process with its applications. **(10 Marks)**
b. With a sketch, explain the electron beam welding. Mention its applications. **(10 Marks)**

7 Write short notes on :
a. Different zones in welding
b. Parameter affecting HAZ
c. Effects of Residual stresses
d. Welding defects. **(20 Marks)**

8 a. Compare the soldering and Brazing processes. **(05 Marks)**
b. What is NDT? With neat sketches, explain the magnetic particle and ultrasonic testing techniques. **(15 Marks)**

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10ME/AU36B

Third Semester B.E. Degree Examination, June/July 2017

Fluid Mechanics

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Distinguish between the following:
i) Mass density and specific weight
ii) Newtonian and non-Newtonian fluid
iii) Absolute and kinematic viscosity **(06 Marks)**

b. Explain the phenomenon of capillarity. Obtain expression for capillary rise of a liquid. **(06 Marks)**

c. Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m × 0.8 m and inclined plane having a inclination of 30°. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of the oil film is 1.5 mm. **(08 Marks)**

2 a. State: i) Pascal's law, ii) Hydro static law. **(02 Marks)**

b. Derive an expression for the total pressure for an inclined force and depth of center of pressure for an inclined surface submerged in water. **(08 Marks)**

c. A simple U tube manometer containing mercury is connected to a pipe in which a fluid of specific gravity 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the center of pipe is 15 cm below. **(05 Marks)**

d. Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the center of the plate is 3m below the free surface of water. Find the position of center of pressure also. **(05 Marks)**

3 a. Define stream function and velocity potential function. **(04 Marks)**

b. The stream function for a two dimensional flow is given by $\Psi = 2xy$ calculate the velocity at the point P(2, 3). Find the velocity potential function ϕ. **(08 Marks)**

c. A block of wood of specific gravity 0.7 floats in water. Determine the meta-centric height of the block if its size is 2m × 1m × 0.8m. **(08 Marks)**

4 a. State Bernoulli's theorem for steady flow of an incompressible fluid and derive an expression for the same. State the assumptions for such a derivation. **(10 Marks)**

b. The water is flowing through a taper pipe of length 100 m having diameters 600 mm at the upper end and 300 mm at the lower end, at the rate of 50 liters/s. The pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm^2. **(10 Marks)**

PART – B

5 a. Derive an expression for rate of flow through venturimeter. **(08 Marks)**

b. Define notch and classify them. **(04 Marks)**

c. Using Buckingham's π-theorem, show that the velocity through a circular orifice is given by $V = \sqrt{2gH}\,\phi\left[\frac{D}{H}, \frac{\mu}{\rho V H}\right]$ where H is the head causing flow, D is the diameter of the orifice, μ is coefficient of viscosity, ρ is the mass density and g is the acceleration due to gravity. **(08 Marks)**

6 a. Derive Darcy-Weisbach equation and deduce it to Chezy's equation. **(08 Marks)**

b. Define hydraulic gradient line and total energy line. **(04 Marks)**

c. Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m, through which water is flowing at a velocity of 3 m/s using (i) Darcy formula, (ii) Chezy's formula for which C = 60. Take γ for water = 0.01 stoke. **(08 Marks)**

7 a. Prove that the velocity distribution for viscous flow between two parallel plates when both plates are fixed across a section is parabolic in nature. Also sketch the velocity distribution. **(10 Marks)**

b. A fluid of viscosity 0.7 N-S/m^2 and specific gravity 1.3 is flowing through a circular pipe of diameter 100 mm. The maximum shear stress at the pipe wall is given as 196.2 N/m^2, find:
i) The pressure gradient
ii) The average velocity
iii) Reynold's number of the flow. **(10 Marks)**

8 a. Define displacement thickness and momentum thickness. **(04 Marks)**

b. Derive an expression for velocity of sound wave in a fluid. **(10 Marks)**

c. Experiments were conducted in a wind tunnel with a wind speed of 50 km/hour on a flat plate of size 2m long and 1m wide. The density of air is 1.15 kg/m^3. The coefficients of lift and drag are 0.75 and 0.15 respectively. Determine:
i) The lift force
ii) The drag force
iii) The resultant force
iv) Direction of resultant force
v) Power exerted by air on the plate **(06 Marks)**

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MATDIP301

Third Semester B.E Degree Examination, June/July 2017
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Express : $\dfrac{1}{(2+i)^2} - \dfrac{1}{(2-i)^2}$ in the form of a + i b. **(07 Marks)**

b. Find the modulus and amplitude of the complex number $1 - \cos\alpha + i\sin\alpha$. **(06 Marks)**

c. Express the complex number $\sqrt{3}+i$ in the polar form. **(07 Marks)**

2 a. Find the n^{th} derivative of $\log(ax + b)$. **(07 Marks)**

b. Find the n^{th} derivative of $\dfrac{x}{(x-1)(2x+3)}$. **(06 Marks)**

c. If $y = \sin^{-1} x$, prove that : $(1 - x^2)y_{n+2} - (2n + 1)x\, y_{n+1} - n^2 y_n = 0$. **(07 Marks)**

3 a. Using Taylor's theorem, expand sin x in power of $(x - \pi/2)$. **(07 Marks)**

b. Obtain the Maclaurin's series expansion of the function $\sqrt{1+\sin 2x}$ up to the term containing x^4. **(06 Marks)**

c. State and prove Euler's theorem. **(07 Marks)**

4 a. Find the total derivative of $z = xy^2 + x^2y$ where $x = at$, $y = 2at$, and also verify the result by direct substitution. **(07 Marks)**

b. If $u = f(y - z, z - x, x - y)$ prove that : $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$. **(06 Marks)**

c. if $x = u(1 - v)$ and $y = uv$, find $J = \dfrac{\partial(x,y)}{\partial(u,v)}$ and $J' = \dfrac{\partial(u,v)}{\partial(x,y)}$ and also verify $J \cdot J' = 1$. **(07 Marks)**

5 a. Obtain the reduction formula for $\int \cos^n x \cdot dx$. **(07 Marks)**

b. Evaluate : $\displaystyle\int_0^2 \frac{x^4}{\sqrt{4-x^2}} \cdot dx$. **(06 Marks)**

c. Evaluate : $\displaystyle\int_1^2\int_1^3 xy^2\, dx\, dy$. **(07 Marks)**

6 a. Evaluate : $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} x\,y\,z\, dz\, dy\, dx$. **(07 Marks)**

b. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. **(06 Marks)**

c. Prove that $\beta(m,n) = \dfrac{\Gamma_m \Gamma_n}{\Gamma(m+n)}$. **(07 Marks)**

7 a. Solve : $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$. (07 Marks)

b. Solve $x^2y\,dx - (x^3 + y^3)\,dy = 0$. (06 Marks)

c. Solve $\frac{dy}{dx} + y\cot x = \cos x$. (07 Marks)

8 a. Solve : $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 4y = 0$. (05 Marks)

b. Solve $\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 9y = 3e^{-4x}$. (05 Marks)

c. Solve : $y'' + 2y' + y = e^{-x} + \cos 2x$. (05 Marks)

d. Solve : $\frac{d^2y}{dx^2} - 4y = x\sin 2x$. (05 Marks)

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